Optimal Receiver for Cooperative Systems
OFDM with Power Amplifier Nonlinear

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Abstract

- Cooperation diversity and orthogonal frequency division multiplexing (OFDM) are two key technologies for the future wireless communication systems.

- We develop the optimum receiver, in the signal-to-noise ratio (SNR) sense, for a nonlinear OFDM cooperative system.

- The system model considered includes a transmitter with a nonlinear power amplifier and an amplify-and-forward (AF) relay also using a nonlinear power amplifier.

- The proposed receiver uses a maximum ratio combining (MRC) diversity technique to treat the received signals, assuming that both the direct link (source-destination) and the relay link are available. Numerical simulation results are provided to illustrate the performance of the proposed receiver.
Motivation

▸ Currently there are few studies that investigate the impact of nonlinear distortions in cooperative communication systems.

▸ No studies in the literature to consider a cooperative system where both source and relay amplifiers are nonlinear and where both links source-destination and source-relay-destination is used.
Modeling System

- The setting of the proposed system has three basic components: source (S), relay (R) and destination (D).
- Where it is assumed that the source and relay PAs are non-linear, as shown in Figure 1.
Linearization System

Let $s_n$ be the information symbol to be transmitted in the $n$-th subcarrier frequency domain, with $1 \leq n \leq N$, where $N$ is the number of subcarriers, and $s'^n$ signal transmitted in the time domain.

For a high number $N$ of subcarriers, $s'^n$ can be considered a random variable with complex Gaussian distribution.
Output of the PA Source

- Assuming that the waveform is rectangular and using the theorem of Bussgang for input complex Gaussian, we can express the output of the PA source in the time domain as:

\[ f'_s (s'_n) = \alpha_s s'_n + d'_n^{(S)}, \]

- In the frequency domain:

\[ f_n^S = \alpha_s s_n + d_n^{(S)}, \]
Output of the PA Relay

- Similarly, we can express the n-th subcarrier of the relay PA in the frequency domain as:

\[ f_n^R = \alpha_r(g_n x_n^{(SD)}) + d_n^{(R)}, \]

- Considered that the relay adds a variable gain \( g_n \) for each subcarrier of the received signal, where:

\[ g_n = \frac{\sqrt{P_r}}{\sqrt{|h_n^{(SR)}|^2 P_s + \sigma^2 \eta_n^{(SR)}}}. \]
Baseband signal into discrete frequency domain, the n-th subcarrier

- Link Source-Relay (S-R):
  \[ x_n^{(SR)} = h_n^{(SR)} \alpha_s s_n + h_n^{(SR)} d_n^{(S)} + \eta_n^{(SR)}. \]

- Link Source-Relay-Destination (S-R-D):
  \[ x_n^{(SRD)} = h_n^{(RD)} h_n^{(SR)} \alpha_s \alpha_r g_n s_n + h_n^{(SR)} h_n^{(RD)} \alpha_r g_n d_n^{(S)} + \alpha_r g_n \eta_n^{(SR)} + h_n^{(RD)} d_n^{(R)} + \eta_n^{(RD)}. \]

- Link Source-Destination (S-D):
  \[ x_n^{(SD)} = h_n^{(SD)} \alpha_s s_n + h_n^{(SD)} d_n^{(S)} + \eta_n^{(SD)}, \]
The signals received at the destination can be expressed as:

\[
\begin{align*}
    x_n^{(SRD)} &= h_{1n}s_n + \nu_{1n} \\
    x_n^{(SD)}  &= h_{2n}s_n + \nu_{2n}
\end{align*}
\]

Where,

\[
\begin{align*}
    h_{1n} &= h_n^{(RD)} h_n^{(SR)} \alpha_s \alpha_r g_n, \\
    h_{2n} &= h_n^{(SD)} \alpha_s, \\
    \nu_{1n} &= h_n^{(SR)} h_n^{(RD)} \alpha_r g_n d_n^{(S)} + h_n^{(RD)} \alpha_r g_n \eta_n^{(SR)} + h_n^{(RD)} g_n d_n^{(R)} + \eta_n^{(RD)}, \\
    \nu_{2n} &= h_n^{(SD)} d_n^{(S)} + \eta_n^{(SD)}.
\end{align*}
\]
Optimal Receiver

- We will use the MRC to combine the two components of the signal received at the destination and to recover the transmitted information.

- However, to achieve this type of receptor is required that the noise is uncorrelated.

- Thus it is necessary to pre-whitening of the signals.
Pre-whitening

\[
\begin{bmatrix}
  x_n^{(SRD)} \\
  x_n^{(SD)}
\end{bmatrix} = \begin{bmatrix}
  h_{1n} \\
  h_{2n}
\end{bmatrix} s_n + \begin{bmatrix}
  \nu_{1n} \\
  \nu_{2n}
\end{bmatrix},
\]

- Whitening matrix $A_n$

\[A_n R_{\nu_n} A_n^H = I_2,\]

- Correlation matrix

\[
R_{\nu_n} = \begin{bmatrix}
  E\{|\nu_{1n}|^2\} & E\{\nu_{1n}\nu_{2n}^*\} \\
  E\{\nu_{1n}^*\nu_{2n}\} & E\{|\nu_{2n}|^2\}
\end{bmatrix}.
\]
Pre-whitening

The array of whitening is defined as:

\[ A_n = \Lambda_n^{-\frac{1}{2}} E_n^H \]

Wherein \( E_n \) and \( \Lambda_n \) respectively represent the matrices of eigenvectors and eigenvalues of the matrix \( R_{\nu n} \).

We can define a new system with orthonormal noise:

\[ x_{bn} = A_n x_n = A_n h_n s_n + A_n \nu_n \]
\[ = h_{bn} s_n + \nu_{bn} \]

\[ \nu_{bn} = A_n \nu_n \]
\[ h_{bn} = A_n h_n \]
The optimal receiver for the proposed system is:

\[ y_n = w_n^T x_{bn} \]

Where:

\[ w_n = h_{bn}^*/\|h_{bn}\|^2 \in \mathbb{C}^{2 \times 1}, \]

The SNR at the output of MRC is given by:

\[
SNR = \sigma_s^2 \|h_{bn}\|^2 = \sigma_s^2 h_n^H A_n^H A_n h_n \\
= \sigma_s^2 h_n^H E_n \Lambda_n^{-1} E_n^H h_n \\
= \sigma_s^2 h_n^H R_{\nu}^{-1} h_n.
\]
RESULTS

- We evaluate the performance of the receiver model proposed by computer simulations
The Figure shows the symbol error rate of the receiver MRC, the forward path of the receiver and the receiver path passing through the relay. Considering the SNR equal to 20dB.
Analysis of Figure 1

▸ It is noted that for most of the tested IBO values, the receiver MRC presents a considerable gain as compared with others.

▸ Providing a SER approximately 10 times higher at optimum IBO receptor.

▸ The existence of optimal point is explained by the fact that for high values of IBO, have a lower transmission power. On the other hand to very low values of IBO, the effects of non-linearity of the transmitted signal are emphasized.
In Figure 2, we studied the receptor MRC considering IBOs equal to 1dB, 5dB, 10dB, 15dB and considering linear PAs at source and relay.
Analysis of Figure 2

- We can see that, for low and high values of with IBO and SNR, the receiver MRC is impaired by high nonlinear distortions.

- It shows a considerable improvement in S.E.R. to the extent that the with IBO increases.

- Interestingly, the receiver MRC with IBO fixed in 10dB has a lower SER than the receiver with IBO fixed in 15dB, which shows again that there is a compromise in the choice of signal strength due to nonlinear distortion added to the signal.
Conclusions

The simulation results showed that the proposed system presents a point of optimum IBO, and that from that point there is a deterioration of the received signal.

Which occurs due to the fact that a low IBOs nonlinear distortion is high and wide for IBOs signal power is reduced.
Future Studies

▸ Analysis capacity

▸ Probability of outage

▸ Optimization of the power source and relay

▸ Developed an optimal receiver considering multiple relays